Name:___________________________

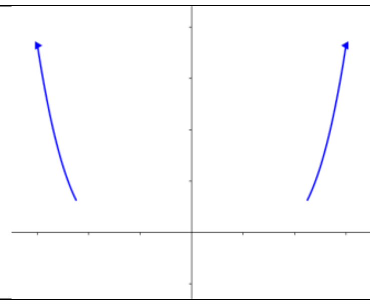
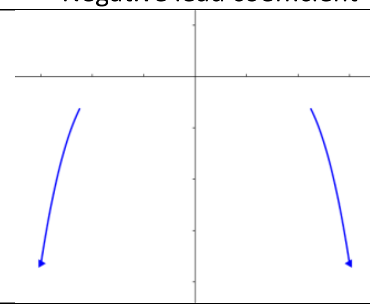
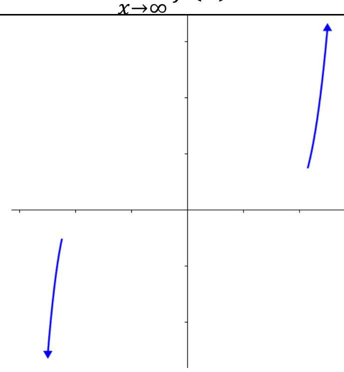
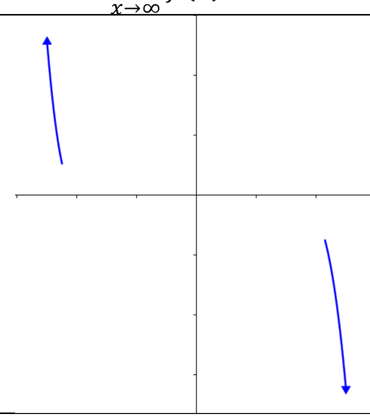
SM3 2.5 Graphing Polynomials

With a known factorization for a polynomial, the roots (x-intercepts) of the polynomial are determined by setting each factor equal to 0 and then solving for the variable. If a factor occurs more than once, it will have a exponent larger than one. We call the number of times that a roots occurs the **multiplicity** of the root. The degree of a polynomial is the sum of the factors' exponents or multiplicities.

Example: Determine the degree of $f(x) = (x-5)^2(x+4)^3$ and roots, with multiplicities, of $f(x)$.

$f(x)$ is a 5th degree polynomial.	Because the sum of the factors' exponents is 5, the degree of the polynomial is 5.
$x = \{5$ w/ m. of 2, -4 w/ m. of $3\}$	Because $x-5=0$ when $x=5$, $x=5$ is a root with multiplicity of 2. Because $x+4=0$ when $x=-4$, $x=-4$ is a root with multiplicity of 3.

The end behavior of a function depends on the degree and lead coefficient of the function.

	Positive lead coefficient	Negative lead coefficient
Even degree Both the left and right end behaviors agree with one another.		
	$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$	$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = -\infty$
Odd degree The left and right end behaviors disagree with one another. Note that each graph's overall slope matches the sign of the lead coefficient.		
	$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$	$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = -\infty$

Graphically, functions cross the x-axis at each root.

- If a root has even multiplicity, the sign of the polynomial on either side of the root will remain unchanged. This means that the function can't pass through the root but must either stay entirely above or below the x-axis on either side of the root.
- If a root has odd multiplicity, the sign of the polynomial on either side of the root will change. This means that the function must pass through the root changing whether it is above or below the x-axis on either side of the root.

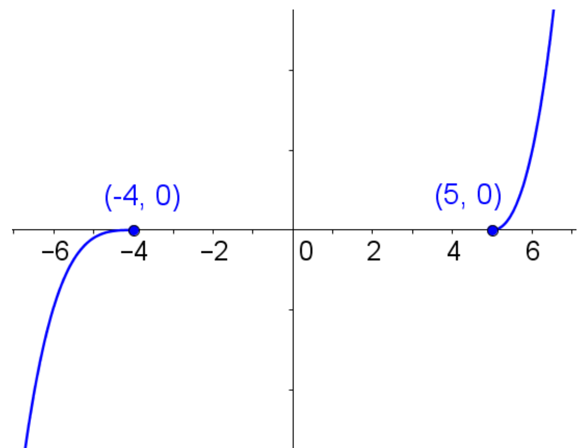
Example: Sketch $f(x) = (x-5)^2(x+4)^3$ and state the end behavior in limit notation.

Step 1) Plot the roots and sketch the function's end behavior.

$f(x)$ has roots at $(-4,0), (5,0)$.

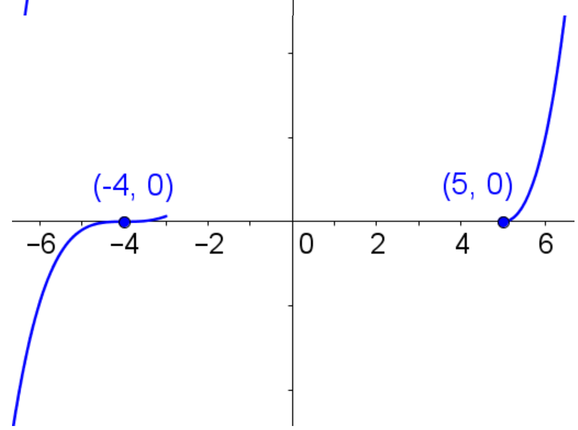
$f(x)$ is a 5^{th} degree polynomial. The lead coefficient is positive.

The end behaviors will disagree. The left end behavior will be low and the right end behavior will be high.

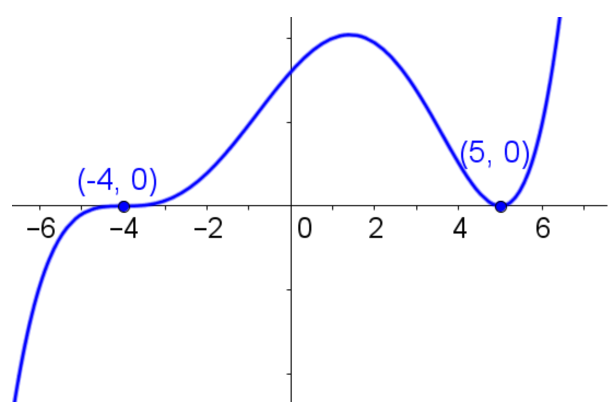


Step 2) The root of $f(x)$ at $(-4,0)$ has multiplicity of 3, which is odd. The function will pass through the root and move from being negative to positive at $x = -4$.

The root of $f(x)$ at $(5,0)$ has multiplicity of 2, which is even. The function will not pass through the root; it will move from being positive to positive at $x = 5$.



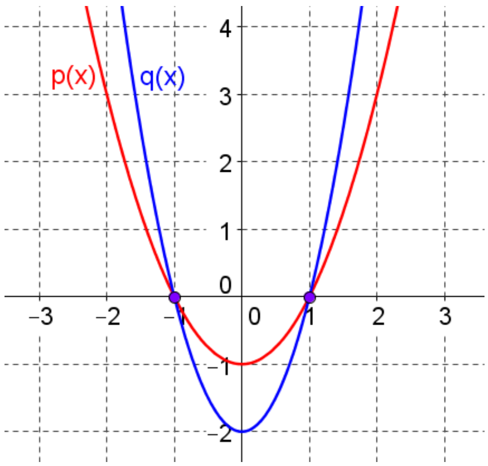
Step 3) Connect the curve of the polynomial between each root. Do not concern yourself with being precise with the height of the curve for every x-value between the roots; just don't create false roots by intersecting the x-axis in any non-root positions.



Step 4) Write the end behavior using limit notation.

$$\lim_{x \to -\infty} f(x) = -\infty, \lim_{x \to \infty} f(x) = \infty$$

Observe that $p(x) = x^2 - 1$ and $q(x) = 2x^2 - 2$ both have the same set of roots, $x = \{-1,1\}$.



The roots of a polynomial do not make the polynomial unique. We could draw many more polynomials that have roots of $x = \{-1,1\}$.

However, if we know all of the roots of the polynomial with the multiplicity of each root, one known non-root value of the polynomial will give us enough information to determine which polynomial we're dealing with.

Example: Find $f(x)$, which has roots $x = \{0,3\}$ and condition $f(1) = 4$

$$\begin{aligned} f(x) &= a(x)(x-3) \\ 4 &= a(1)(1-3) \\ 4 &= -2a \\ -2 &= a \end{aligned}$$

$$f(x) = -2(x)(x-3)$$

$$\boxed{f(x) = -2x^2 + 6x}$$

Write the polynomial's factorization, including a variable to represent lead coefficient. Substitute the values from the condition. Solve for the lead coefficient. Write the polynomial's factorization, including the known lead coefficient. Distribute to find the polynomial.

Example: Find $g(x)$, which has roots $x = \{-4,10\}$ and condition $g(2) = -144$

$$\begin{aligned} g(x) &= a(x \qquad)(x \qquad) \\ &= a(\qquad)(\qquad) \\ &= a(\ )(\ \ ) \\ &= \qquad a \\ &= a \end{aligned}$$

$$g(x) = \ \ (x \qquad)(x \qquad)$$

$$\boxed{g(x) = 3x^2 - 18x - 120}$$

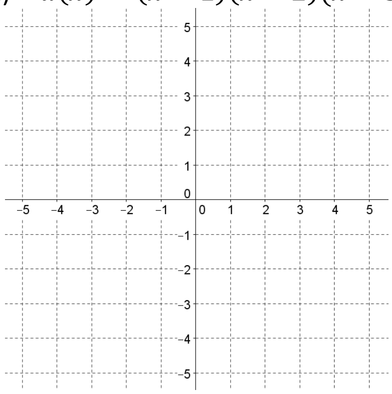
Write the polynomial's factorization, including a variable to represent lead coefficient. Substitute the values from the condition. Solve for the lead coefficient.

Write the polynomial's factorization, including the known lead coefficient. Distribute to find the polynomial.

HW2.5

Sketch the polynomial with accurate roots and end behavior. Discuss the end behavior of the polynomial using limit notation.

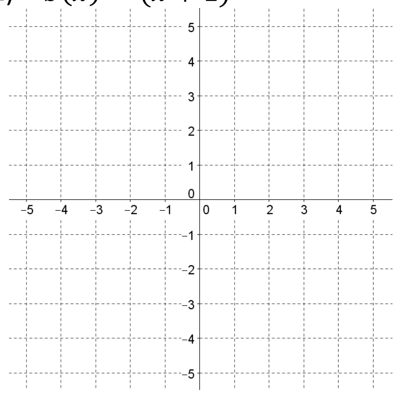
1) $a(x) = (x-1)(x-2)(x-3)$



$\lim_{x \to -\infty} a(x) =$

$\lim_{x \to \infty} a(x) =$

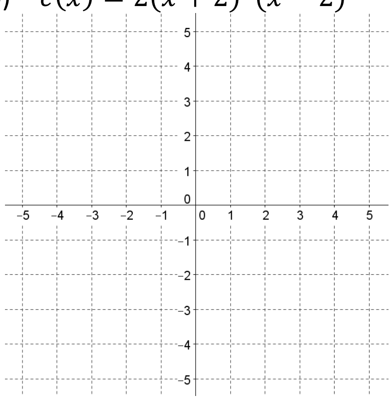
2) $b(x) = (x+1)^3$



$\lim_{x \to -\infty} b(x) =$

$\lim_{x \to \infty} b(x) =$

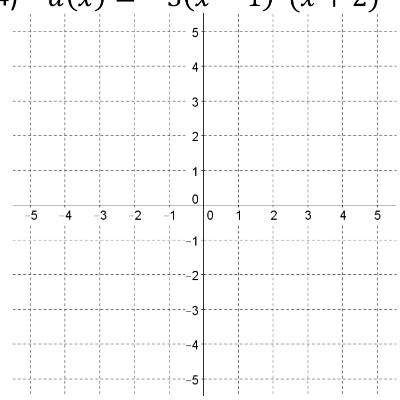
3) $c(x) = 2(x+2)^2(x-2)^2$



$\lim_{x \to -\infty} c(x) =$

$\lim_{x \to \infty} c(x) =$

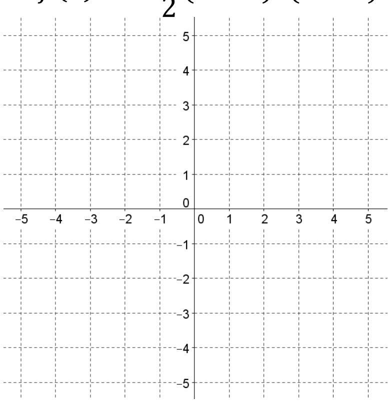
4) $d(x) = -3(x-1)^3(x+2)^2$



$\lim_{x \to -\infty} d(x) =$

$\lim_{x \to \infty} d(x) =$

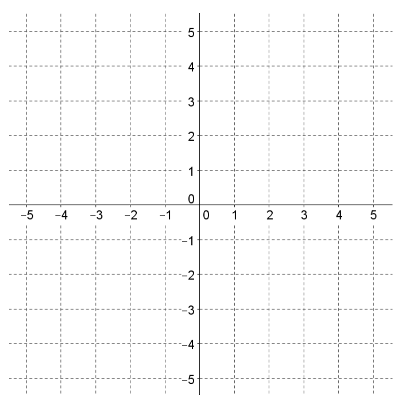
5) $f(x) = -\frac{1}{2}(x+4)^2(x-3)^2$



$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to \infty} f(x) =$

6) $g(x) = x(x-4)^3$



$\lim_{x \to -\infty} g(x) =$

$\lim_{x \to \infty} g(x) =$

State the least degree polynomial, in descending order, that has the given roots and satisfies the condition:

9) $x = \{-2,5\}; f(1) = -36$

10) $x = \left\{-3, \frac{1}{2}\right\}; f(1) = 8$

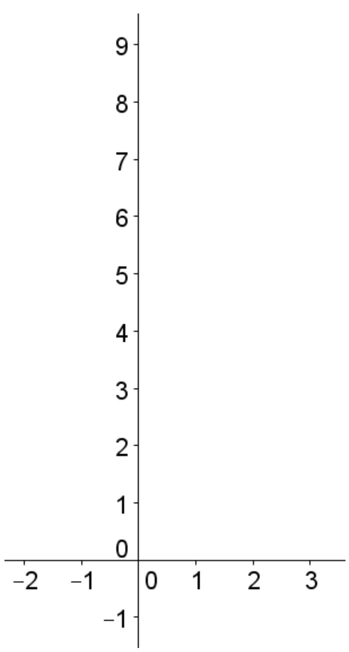
11) $x = \{-5i, 5i\}; f(1) = -52$

12) $x = \left\{-1, -\frac{3}{4}\right\}; f(2) = \frac{33}{2}$

13) $x = \left\{\frac{2}{3}, \frac{3}{2}\right\}; f(-1) = 50$

16) $x = \{-2$ w/ m. of $2, 2\}$; $f(1) = -27$

Cumulative Problem: Build and graph a **cubic** polynomial to meet the following specifications:



Roots: $x = \{-1,2\}$

Increasing interval: $(-\infty, 0) \cup (2, \infty)$
Decreasing interval: $(0,2)$

Condition: $f(0) = 8$

End Behavior:
$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$